دانشگاه صنعتی اصفهان
دانشکده مهندسی حمل و نقل

پژوهش عملیاتی

مسئله حمل و نقل

Transportation Problem

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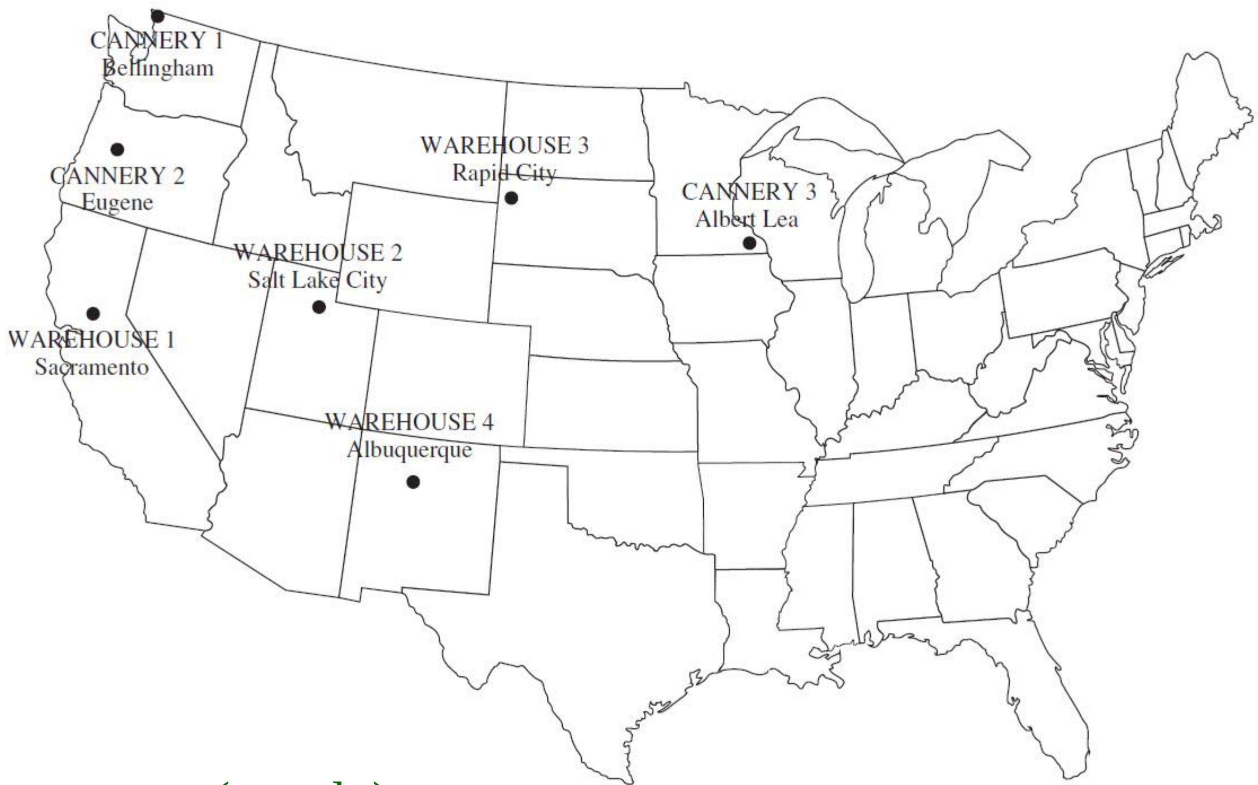
We want to assign shipments to Origin-Destination combinations

An example:
From canneries
To warehouses



Given:

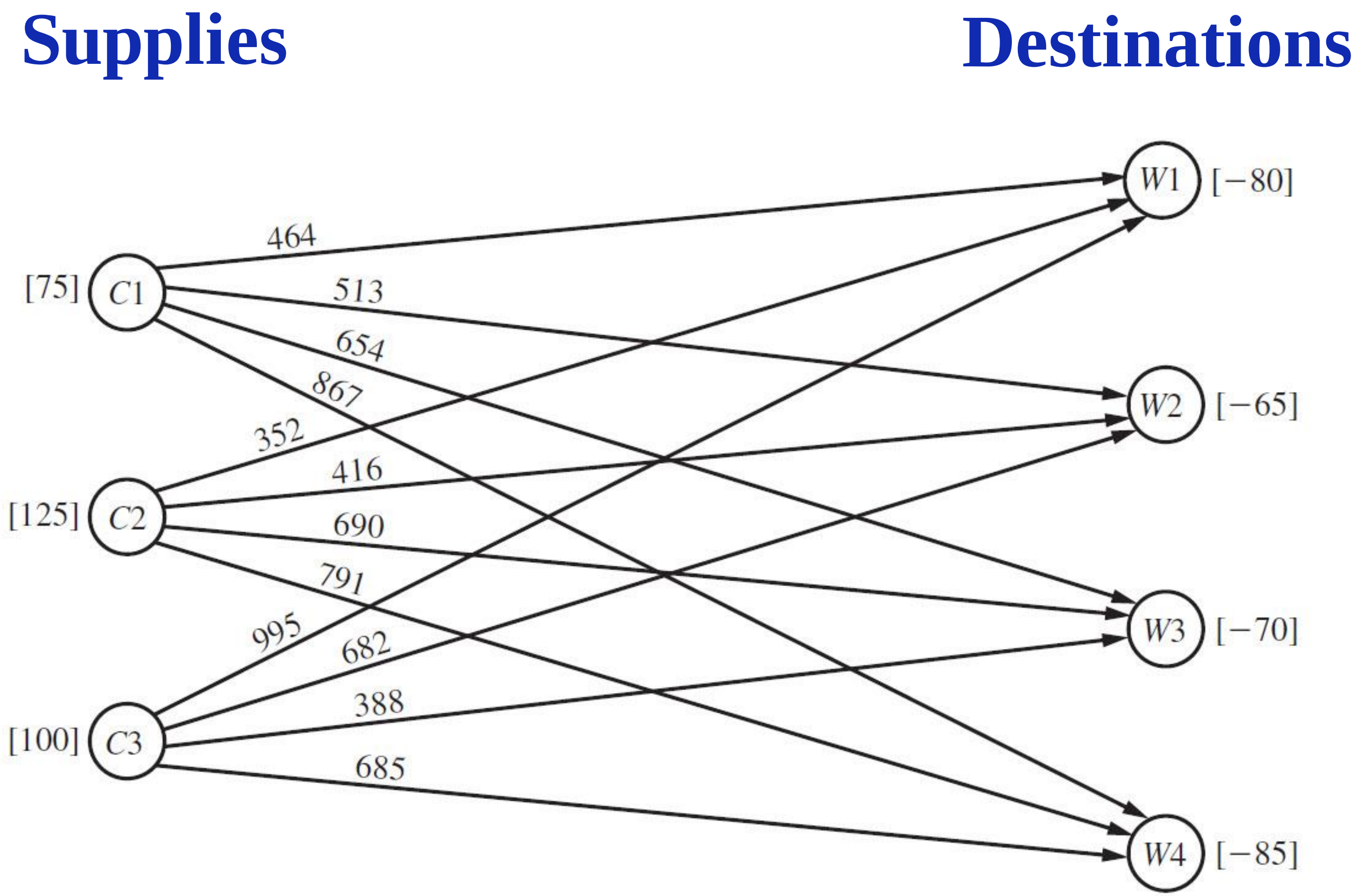
- Total output from each cannery (supply)
- Total input to each warehouse (destination)
- shipping cost per truckload for each cannery-warehouse combination

		Shipping Cost ($) per Truckload				
		Warehouse				
		1	2	3	4	Output
	1	464	513	654	867	75
Cannery	2	352	416	690	791	125
	3	995	682	388	685	100
Allocation		80	65	70	85	

Demands

Supplies

How to represent it as a network ?



Network Representation of Transportation Problem

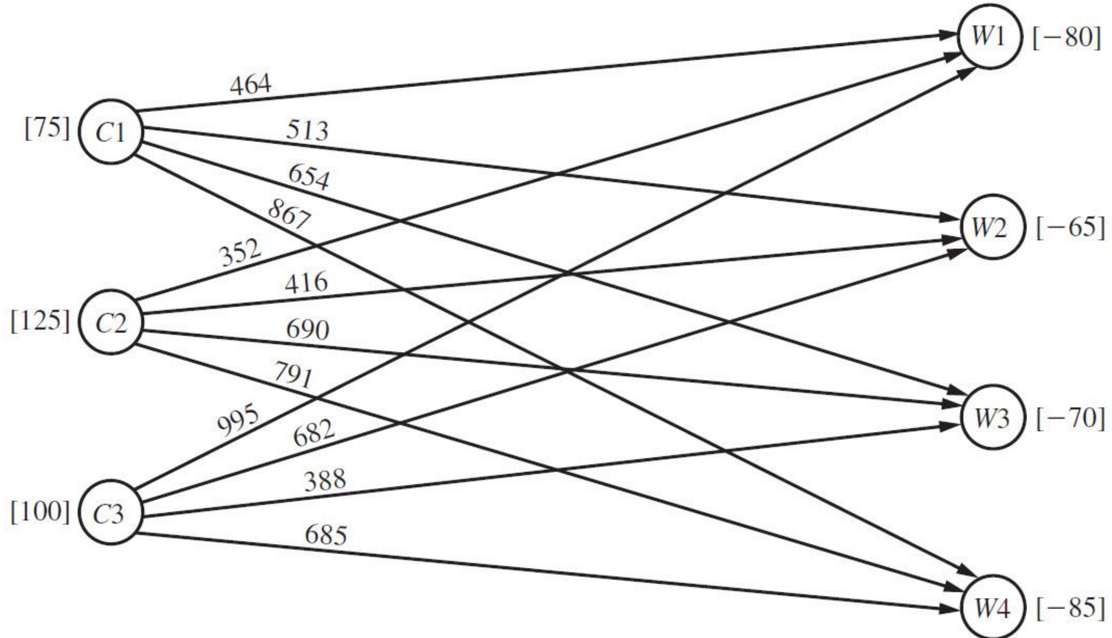
Variables

x_{ij} number of truckloads to be shipped from cannery i to warehouse j (Cont.).

Objective Function

Minimize total cost $\sum_{i,j} c_{ij} x_{ij}$

Minimize $Z = 464x_{11} + 513x_{12} + 654x_{13} + 867x_{14} + 352x_{21} + \ldots + 685x_{34}$



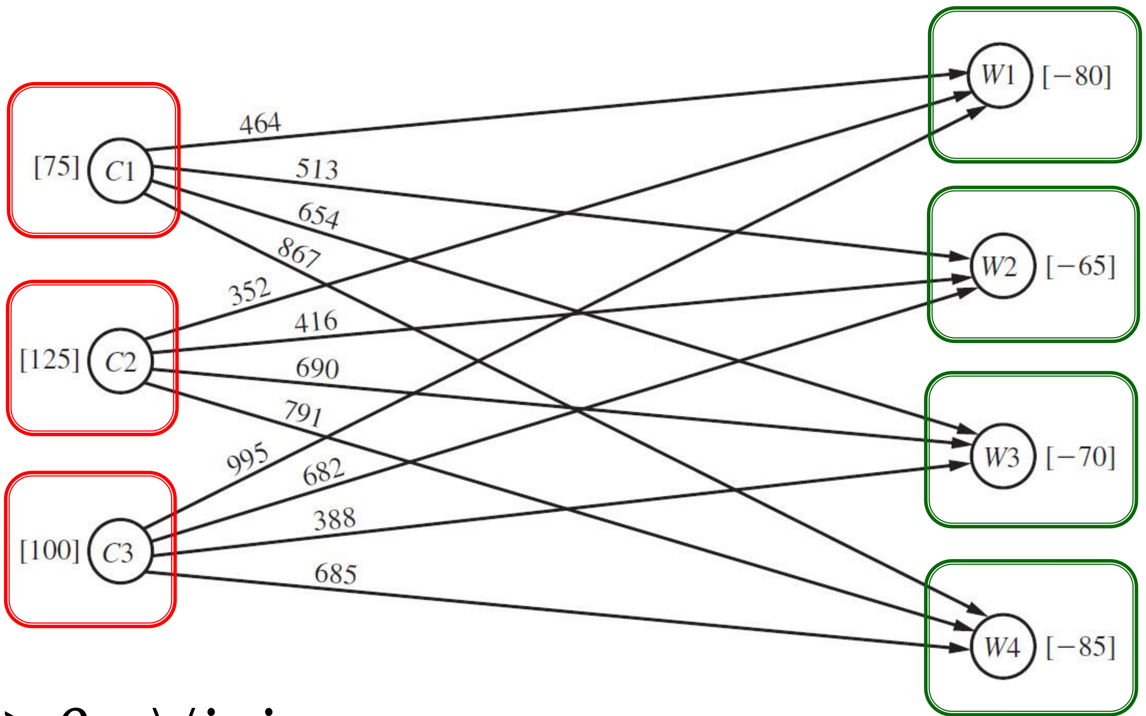
Constraints

Supply Constraints:

$$\sum_{j} x_{ij} = S_i \quad \forall i$$

Demand Constraints:

$$\sum_{i} x_{ij} = D_j \quad \forall j$$



$$x_{ij} \geq 0 \quad \forall i, j$$

Constraints

$$
\left.
\begin{aligned}
x_{11} + x_{12} + x_{13} + x_{14} &= 75 \\
x_{21} + x_{22} + x_{23} + x_{24} &= 125 \\
x_{31} + x_{32} + x_{33} + x_{34} &= 100
\end{aligned}
\right\}
$$

$$
\left.
\begin{aligned}
x_{11} + x_{21} + x_{31} &= 80 \\
x_{12} + x_{22} + x_{32} &= 65 \\
x_{13} + x_{23} + x_{33} &= 70 \\
x_{14} + x_{24} + x_{34} &= 85
\end{aligned}
\right\}
$$

All coefficients: 0 or 1

(Transportation Problem)

Constraints

Coefficient of:

$$
\begin{array}{c|ccccccccccccc}
 & x_{11} & x_{12} & \cdots & x_{1n} & x_{21} & x_{22} & \cdots & x_{2n} & \cdots & x_{m1} & x_{m2} & \cdots & x_{mn} \\
\hline
 & 1 & 1 & \cdots & 1 & & & & & & & & & \\
 & & & & & 1 & 1 & \cdots & 1 & & & & & \\
\text{Supply Constraints} & & & & & & & & & \ddots & & & & \\
 & & & & & & & & & & 1 & 1 & \cdots & 1 \\
\mathbf{A} = & 1 & & & & 1 & & & & & 1 & & & \\
 & & 1 & & & & 1 & & & \cdots & & 1 & & \\
\text{Demand Constraints} & & & \ddots & & & & \ddots & & & & & \ddots & \\
 & & & & 1 & & & & 1 & & & & & 1
\end{array}
$$

Matrix Form

(Transportation Problem)

The requirements assumption

(Equality Constraint) =

- ✓ Each source has a **fixed** supply, **entire** supply must be distributed to destinations.
- ✓ Each destination has a **fixed** demand, **entire** demand must be received from sources.

The feasible solutions property:
A transportation problem will have feasible solutions ⟺ $$\sum_{i=1}^{m} S_i = \sum_{j=1}^{n} D_j$$

The cost assumption

The cost of shipping is **directly proportional** to the number of units distributed.

$$c_{ij} x_{ij}$$

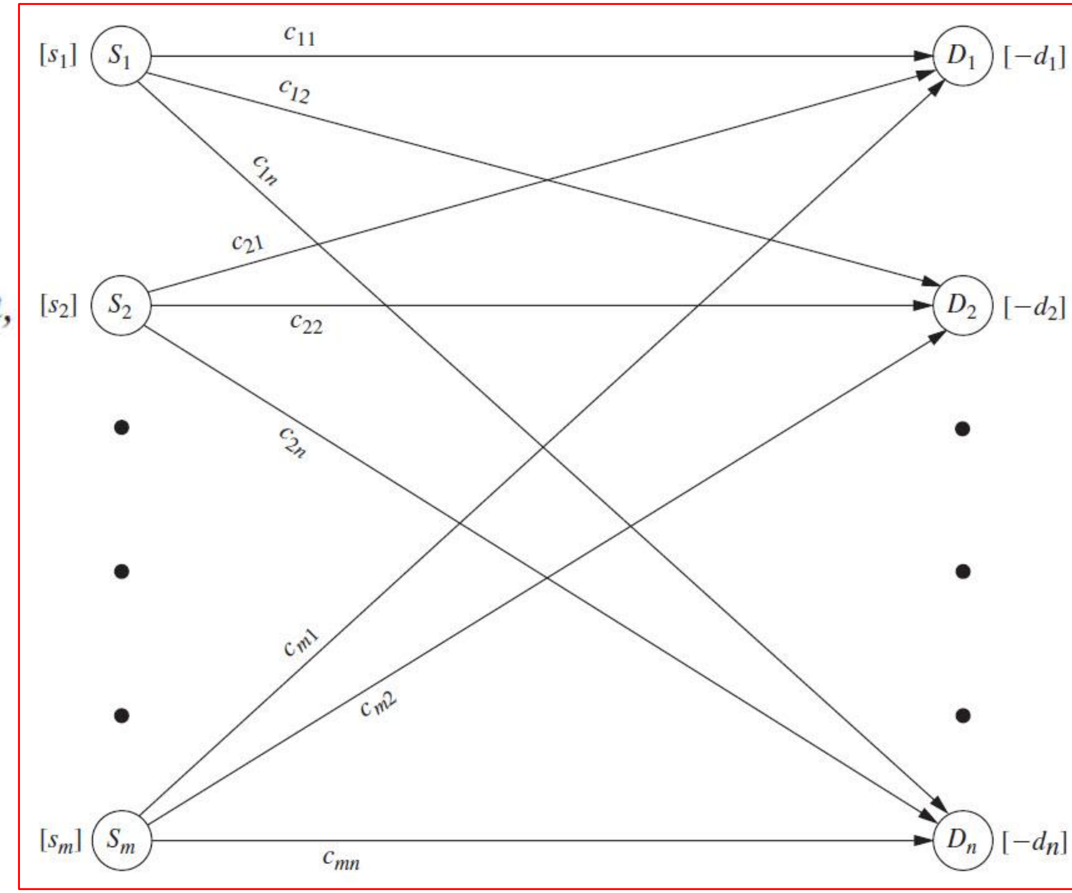
Minimize $$Z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij},$$

Subject to:

$$\sum_{j=1}^{n} x_{ij} = s_i \qquad \text{for } i = 1, 2, \ldots, m,$$

$$\sum_{i=1}^{m} x_{ij} = d_j \qquad \text{for } j = 1, 2, \ldots, n,$$

$$x_{ij} \geq 0, \qquad \text{for all } i \text{ and } j.$$



Type:
Linear Programming (LP)

In Transportation Problem:
Every s_i and d_j have an integer value

All the basic variables (non-zero,allocations)
in every basic feasible (BF) solution (including an optimal one)
also have integer values.

Transportation Problem

	A	B	C	D	E	F	G	H	I	J
1	P&T Co. Distribution Problem									
2										
3		Unit Cost		Destination (Warehouse)						
4				Sacramento	Salt Lake City	Rapid City	Albuquerque			
5		Source	Bellingham	$464	$513	$654	$867			
6		(Cannery)	Eugene	$352	$416	$690	$791			
7			Albert Lea	$995	$682	$388	$685			
8										
9										
10		Shipment Quantity		Destination (Warehouse)						
11		(Truckloads)		Sacramento	Salt Lake City	Rapid City	Albuquerque	Total Shipped		Supply
12		Source	Bellingham	0	20	0	55	75	=	75
13		(Cannery)	Eugene	80	45	0	0	125	=	125
14			Albert Lea	0	0	70	30	100	=	100
15			Total Received	80	65	70	85			
16				=	=	=	=			Total Cost
17			Demand	80	65	70	85			$ 152,535

Changing cells

objective value

Solver Parameters

Set Target Cell: TotalCost

Equal To: ○ Max ◉ Min ○

By Changing Cells:

ShipmentQuantity

Subject to the Constraints:

TotalReceived = Demand
TotalShipped = Supply

Solver Options

☑ Assume Linear Model

☑ Assume Non-Negative

Range Name	Cells
Demand	D17:G17
ShipmentQuantity	D12:G14
Supply	J12:J14
TotalCost	J17
TotalReceived	D15:G15
TotalShipped	H12:H14
UnitCost	D5:G7

	H
11	Total Shipped
12	=SUM(D12:G12)
13	=SUM(D13:G13)
14	=SUM(D14:G14)

	C	D	E	F	G
15	Total Received	=SUM(D12:D14)	=SUM(E12:E14)	=SUM(F12:F14)	=SUM(G12:G14)

	J
16	Total Cost
17	=SUMPRODUCT(UnitCost,ShipmentQuantity)

some problems violate the requirements assumption: not quite fit as a transportation problem

Reformulation: introducing a dummy destination or a dummy source.

Example: Airplane Company

Production of jet engines
+
Installation of jet engines

Month	Scheduled Installations	Maximum Production	Unit Cost* of Production	Unit Cost* of Storage
1	10	25	1.08	0.015
2	15	35	1.11	0.015
3	25	30	1.10	0.015
4	20	10	1.13	

How much production in each month?

Month	Scheduled Installations	Maximum Production	Unit Cost* of Production	Unit Cost* of Storage
1	10	25	1.08	0.015
2	15	35	1.11	0.015
3	25	30	1.10	0.015
4	20	10	1.13	

Cost

Minimize: Cost of Production + Cost of storage

Variables

x_j Number of jet engines to be produced in month j, for j=1, 2, 3, 4.

z_j Cumulative number of jet engines to be stored until month j, for j=1, 2, 3, 4.

Let $z_1 = x_1 - 10,$

$$z_2 = x_1 + x_2 - 25,$$

$$z_3 = x_1 + x_2 + x_3 - 50,$$

$$z_4 = x_1 + x_2 + x_3 + x_4 - 70.$$

Month	Scheduled Installations	Maximum Production	Unit Cost* of Production	Unit Cost* of Storage
1	10	25	1.08	0.015
2	15	35	1.11	0.015
3	25	30	1.10	0.015
4	20	10	1.13	

Cost

Objective Function

minimize $1.08x_1 + 1.11x_2 + 1.10x_3 + 1.13x_4 + 0.15(z_1 + z_2 + z_3 + z_4)$

Constraints

$$x_1 - z_1 = 10$$
$$x_1 + x_2 - z_2 = 25$$
$$x_1 + x_2 + x_3 - z_3 = 50$$
$$x_1 + x_2 + x_3 + x_4 - z_4 = 70$$
$$0 \le x_1 \le 25$$
$$0 \le x_2 \le 35$$
$$0 \le x_3 \le 30$$
$$0 \le x_4 \le 10$$
$$z_1, z_2, z_3, z_4 \ge 0$$

A linear Programming
But
Not as a transportation problem

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A linear Programming

As a transportation problem

Month	Scheduled Installations	Maximum Production	Unit Cost* of Production	Unit Cost* of Storage
1	10	25	1.08	0.015
2	15	35	1.11	0.015
3	25	30	1.10	0.015
4	20	10	1.13	

- ✓ **Supply i: Production of of jet engines in month i (i = 1, 2, 3, 4)** ?
- ✓ **Demand j: Installation of of jet engines in month j (j = 1, 2, 3, 4)** Given

Variables

x_{ij} Number of jet engines produced in month i for installation in month j

Month	Scheduled Installations	Maximum Production	Unit Cost* of Production	Unit Cost* of Storage
1	10	25	1.08	0.015
2	15	35	1.11	0.015
3	25	30	1.10	0.015
4	20	10	1.13	

Objective Function $Minimize \quad \sum c_{ij} x_{ij}$

c_{ij}

		Cost per Unit Distributed				
		Destination				
		1	2	3	4	Supply
Source	1	1.080	1.095	1.110	1.125	?
	2	?	1.110	1.125	1.140	?
	3	?	?	1.100	1.115	?
	4	?	?	?	1.130	?
Demand		10	15	25	20	

No real cost ($x_{ij} = 0$) → c_{ij} = M (to force x_{ij} to be 0)

Constraints

$$x_{11} + x_{12} + x_{13} + x_{14} \le 25$$
$$x_{21} + x_{22} + x_{23} + x_{24} \le 35$$
$$x_{31} + x_{32} + x_{33} + x_{34} \le 30$$
$$x_{41} + x_{42} + x_{43} + x_{44} \le 10$$

Not in form of Transportation Planning

Use "Slack Variables" As Dummy Destination

Dummy Destination: represents the unused production capacity

Demand for Dummy Destination: total unused production capacity

$$\text{Demand}_{D.D.} = (25 + 35 + 30 + 10) - (10 + 15 + 25 + 20) = 30$$

c_{ij}

		Cost per Unit Distributed					
		Destination					
		1	2	3	4	5(*D*)	Supply
Source	1	1.080	1.095	1.110	1.125	0	25
	2	*M*	1.110	1.125	1.140	0	35
	3	*M*	*M*	1.100	1.115	0	30
	4	*M*	*M*	*M*	1.130	0	10
Demand		10	15	25	20	30	

There is no cost incurred by a fictional allocation.

Demand $_{\text{D.D.}}$

Cost entries of M would be inappropriate for this column because we do not want to force the corresponding values of x_{ij} to be zero. In fact, these values need to sum to 30.

Some problems may fail to satisfy model for transportation problem:

1 distribution does not go directly from sources to destinations
(passes through transfer points) → Transshipment Problem

2 Transshipment Problem +
Upper limits for shipping lanes → Minimum Cost Flow Problem

3 Violating "cost assumption"
Non-linear function of the number of units distributed

4 Violating "requirements assumption"
S_i or D_j are not fixed

For example, final demand at a destination may not become known until after units have arrived.
Then, a nonlinear cost is incurred if amount received deviates from the final demand.

Case 8.1 Shipping wood to Market

(Book: Hillier 2010, Page 356)

Use Excel Solver

CASES

CASE 8.1 Shipping Wood to Market

Alabama Atlantic is a lumber company that has three sources of wood and five markets to be supplied. The annual availability of wood at sources 1, 2, and 3 is 15, 20, and 15 million board feet, respectively. The amount that can be sold annually at markets 1, 2, 3, 4, and 5 is 11, 12, 9, 10, and 8 million board feet, respectively.

In the past the company has shipped the wood by train. However, because shipping costs have been increasing, the alternative of using ships to make some of the deliveries is being investigated. This alternative would require the company to invest in some ships. Except for these investment costs, the shipping costs in thousands of dollars per million board feet by rail and by water (when feasible) would be the following for each route:

	Unit Cost by Rail ($1,000's) Market					Unit Cost by Ship ($1,000's) Market				
Source	1	2	3	4	5	1	2	3	4	5
1	61	72	45	55	66	31	38	24	—	35
2	69	78	60	49	56	36	43	28	24	31
3	59	66	63	61	47	—	33	36	32	26

F. Hillier, G. J. Lieberman, “Introduction to Operations Research”, Ninth Edition, 2010.